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Application of the R-Functions Theory to Problems of Nonlinear Dynamics of Laminated Composite Shallow Shells and Plates: Review

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Abstract
A review of studies performed using the R-functions theory to solve problems of nonlinear dynamics of plates and shallow shells is presented The systematization of results and studies for the problems of free and parametric vibrations and for problems of static and dynamic stability is fulfilled Expansion of the developed original method of discretization for nonlinear movement equations on new classes of nonlinear problems is shown. These problems include researches of vibrations of antisymmetric laminated cylindrical and spherical panels; laminated composite shallow shells with variable thicknesss of the layers; functionally graded (FG) shallow shells and others. The basic issues that arise when using RFM are described
The future prospects of using the theory of R-functions for solving problems of nonlinear dynamics of plates and shallow shells with complex form are formulated First of all this is an algorithms development and creation of the associated software to apply multi-modes approximations; improvement of approximation tools for nonlinear problems; investigation of the cracked functionally graded shallow shells; FG panels under thermal environments; parametric vibrations, static and dynamical stability of the multilayered and FG plates and shells.

Keywords
R-functions theory, laminated composite, shallow shells and plates, nonlinear vibrations, parametrical vibrations, dynamic stability .

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Introduction

The R-functions theory was created by Vladimir L. Rvachev in 70-80th of the last century. This has allowed to overcome one of the major challenges of dynamic problems of the theory of plates and shells, connected with the basic functions construction in the case of an arbitrary domain. Note, these systems have been represented in form of a single analytical expression. Dynamic problems of the plates and shells theory were among the first applications of the method of R-functions. Later this method received a worldwide recognition and nowadays it is known as a method of RFM (R-functions method). Let us present a short survey of the RFM application to vibration problems of the plates and shallow shells.

1. Linear vibrations of plates and shallow shells

Efficiency and universality of the RFM have been illustrated on a large number of linear vibrations problems and stability of plates with complex forms and various boundary conditions. Results obtained for these classes of problems in the period from 1967 to 1998 have been (were) reflected in the monographs [1-4] and in a review article of V.Rvachev and L.Kurpa [5].

Since 1999, the method of R-functions got an intensive application to the problems of free vibrations of isotropic and then orthotropic and multi-layered shallow shells [6-18]. To solve this class of problems the classical theory of shallow shells (CST) was used. This theory is based on the hypotheses of Kirchhoff-Love. The new solution structures were built for different types of boundary conditions. In particular, in Ref. [6, 7] the new structural formula was proposed. Here a crack at the free edge of the plate or shell was taken into account. In Ref. [6-18] a lot of numerical results for the natural frequencies and modes of plates and shallow shells with complex planforms have been (were) presented. In particular, thin plates and shallow shells of variable thickness were studied in Ref. [9, 18]. The thickness was varied according to different laws (linear in one and two directions, exponential, etc.). The investigations were carried out in a framework of the developed software the on base of POLE-RL system.

It should be noted, that until 2000 the theory of R-functions was applied to the study of motion equations of plates and shallow shells in the framework of classical theory. Soon after application of the RFM to the study of laminated composite shells of a moderate thickness, the refined shear deformation theory of the first order (FSDT) has been used. In the papers [14-18] the RFM was developed to solve the problems of vibrations of multi-layered plates in the framework of the refined theory. The results of research carried out in the period from 1997 to 2009 were summarized and presented in the monograph of L.Kurpa [18].

2. Geometrically nonlinear vibrations of plates and shallow shells

Starting of 1999, the first attempt was made to apply the method of R-functions for geometrically nonlinear vibrations of isotropic and later of orthotropic plates of arbitrary planforms. A review of literature showed that numerical results in this area are only available for rectangular plates with a fairly limited set of boundary conditions. The methods used in these studies are based on classical ideas of variables separation with respect to time and coordinates (x, y). With this purpose the expansion of the unknown functions in a limited number of Fourier series is used. Coefficients of the expansion depend on time. However, in the case of a complex geometry and boundary conditions that differ from simply supported ones, this expansion is not so easily applicable, due to the difficulty of a system of basic functions construction that satisfy the given boundary conditions. Here, the most pronounced advantages of the R-functions theory, allowing to construct such basic functions in an analytical form go forward.

One of the first papers applying RFM to study stability of isotropic plate of an arbitrary form are Ref. [40-44]. To solve this problem one-and-two-mode approximation of unknown functions with respect to time has been used. For the mathematical formulation of the problem the classical theory of plates based on the hypotheses of Kirchhoff-Love has been applied. The proposed approach allowed to reduce the nonlinear system of differential equations with partial derivatives to the nonlinear system of two ordinary differential equations. The solution of the obtained system was obtained by the method of Runge-Kutta. In Ref. [20-22] this approach was applied to the orthotropic plates. In Ref [23, 26] the further development of the proposed approach for isotropic shallow shells and plates was presented.

3. Laminated composite plates and shallow shells

Since 2000, the method of R-functions is actively beginning to be applied to the study of geometrically nonlinear vibrations of laminated composite plates and shallow shells [24, 25, 32, 33]. In this case the mathematical formulation of the problem in the framework of two theories is made: classical and refined of the first order theory, which takes into account shear deformations and inertia of rotation of the shells. The original idea of discretization of the nonlinear system of motion equations was proposed in Ref. [27]. The first time analytical formulas were obtained for laminated composite shells of the symmetrical structure provided multimode approximation of the unknown functions. Let us consider the main stages of this approach.

-The first step is to find the eigenfunctions and eigenvalues of the linear oscillations of a shallow shell. This problem is solved by RFM.

-The second step is solving the sequence of inhomogeneous PDE systems. The right-hand parts of these systems are determined by their eigenfunctions.

- The third step is forming the required solution. If the laminated composite shells have symmetrical structures, then the unknown functions are presented as:

$$w=\sum_{i=1}^{n}y_i(t)w_i^{(c)}(x,y),\quad \psi_x=\delta\sum_{i=1}^{n}y_i(t)\psi_{xi}^{(c)}(x,y),\quad \psi_y=\delta\sum_{i=1}^{n}y_i(t)\psi_{yi}^{(c)}(x,y)$$

$$u=\sum_{i=1}^{n}y_i(t)u_i^{(c)}(x,y)+\sum_{i=1}^{n}\sum_{j=1}^{n}y_iy_ju_{ij}\ ,\ v=\sum_{i=1}^{n}y_i(t)v_i^{(c)}(x,y)+\sum_{i=1}^{n}\sum_{j=1}^{n}y_iy_jv_{ij}\quad ,\qquad (1)$$

where $y_k(t)$ are unknown functions in time, $w_i^{(c)}(x,y)$, $u_i^{(c)}(x,y)$, $v_i^{(c)}(x,y)$, $\psi_{xi}^{(c)}(x,y)$, $\psi_{yi}^{(c)}(x,y)$ are components of the i-th eigenfunctions of linear vibrations of the shell. Indicator δ is the tracing constant which takes values of 1 and 0 for the FSDT and CST respectively. The functions u_{ij},v_{ij} must be the solutions of the following system [9]

$$\begin{cases}L_{11}u_{ij}+L_{12}v_{ij}=-Nl_1^{(2)}(w_i,w_j)\\ L_{21}u_{ij}+L_{22}v_{ij}=-Nl_2^{(2)}(w_i,w_j)\end{cases}\qquad (2)$$

But if multi-layered shells have asymmetric structure and FSDT is applied, then the unknown functions u,v,w,ψ_x,ψ_y must be presented as:

$$\begin{cases}u(x,y,t)=\sum_{i=1}^{n}y_i(t)u_i^{(c)}(x,y)+\sum_{i=1}^{n}\sum_{j=1}^{n}y_i(t)y_j(y)u_{ij}(x,y)\\ v(x,y,t)=\sum_{i=1}^{n}y_i(t)v_i^{(c)}(x,y)+\sum_{i=1}^{n}\sum_{j=1}^{n}y_i(t)y_j(y)v_{ij}(x,y)\\ w(x,y,t)=\sum_{i=1}^{n}y_i(t)w_i^{(c)}(x,y)\\ \psi_x(x,y,t)=\sum_{i=1}^{n}y_i(t)\psi_{xi}^{(c)}(x,y)+\sum_{i=1}^{n}\sum_{j=1}^{n}y_i(t)y_j(y)\psi_{xij}(x,y)\\ \psi_y(x,y,t)=\sum_{i=1}^{n}y_i(t)\psi_{yi}^{(c)}(x,y)+\sum_{i=1}^{n}\sum_{j=1}^{n}y_i(t)y_j(y)\psi_{yij}(x,y)\end{cases}\qquad (3)$$

Coefficients of this expansion are the functions $y_i(t)$ depending on time also. But the functions $u_{ij},v_{ij},\psi_{xij},\psi_{yij}$ must be the solutions of the following system of differential equations:

$$\begin{cases}L_{11}u_{ij}(x,y)+L_{12}v_{ij}(x,y)+L_{14}\psi_{xij}(x,y)+L_{15}\psi_{yij}(x,y)=NL_1\left(w_i^{(c)},w_j^{(c)}\right)\\ L_{21}u_{ij}(x,y)+L_{22}v_{ij}(x,y)+L_{24}\psi_{xij}(x,y)+L_{25}\psi_{yij}(x,y)=NL_2\left(w_i^{(c)},w_j^{(c)}\right)\\ L_{41}u_{ij}(x,y)+L_{42}v_{ij}(x,y)+L_{44}\psi_{xij}(x,y)+L_{45}\psi_{yij}(x,y)=NL_4\left(w_i^{(c)},w_j^{(c)}\right)\\ L_{51}u_{ij}(x,y)+L_{52}v_{ij}(x,y)+L_{54}\psi_{xij}(x,y)+L_{55}\psi_{yij}(x,y)=NL_5\left(w_i^{(c)},w_j^{(c)}\right)\end{cases}\qquad (4)$$

Expressions for the linear operators $L_{rs},r,s=\overline{1,5}$ coincide with the corresponding ones, presented in Ref. [18]. The nonlinear expressions $Nl_1^{(2)}(w_i,w_j)$, $Nl_2^{(2)}(w_i,w_j)$, $Nl_4^{(2)}(w_i,w_j)$, $Nl_5^{(2)}(w_i,w_j)$ are defined as:

$$NL_1(w)=-L_{11}(w)\frac{\partial w}{\partial x}-L_{12}(w)\frac{\partial w}{\partial y},\ NL_2(w)=-L_{12}(w)\frac{\partial w}{\partial x}-L_{22}(w)\frac{\partial w}{\partial y},$$

$$NL_4(w)=-L_{41}(w)\frac{\partial w}{\partial x}-L_{42}(w)\frac{\partial w}{\partial y},\ NL_5(w)=-L_{42}(w)\frac{\partial w}{\partial x}-L_{44}(w)\frac{\partial w}{\partial y}.$$

Obtained systems (2) and (4) supplemented by corresponding boundary conditions are solved by RFM. Substituting expressions (1) or (3) for the functions u,v,w,ψ_x,ψ_y in the initial system of motion equations and applying the Bubnov-Galerkin procedure we obtain a nonlinear system of ordinary differential equations in unknown functions $y_j(t)$, of the following type:

$$y_j''(t)+\alpha_jy_j(t)+\sum_{i=1}^{n}\sum_{k=1}^{n}\beta_{jik}y_i(t)y_k(t)+\sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}\gamma_{jikl}y_i(t)y_k(t)y_l(t)=\widetilde{F}\quad \left(j=\overline{1,n}\right)\qquad (5)$$

Expressions for the coefficients $\alpha_j, \beta_{jik}, \gamma_{jikl}$ are determined and expressed through double integrals of known functions [27]. The solution of the obtained system (5) can be performed by different approximate methods, such as the harmonic balance method (HBM), multiscale, the method of Runge-Kutta, Bubnov-Galerkin and others.

The proposed method was applied for studying nonlinear free and forced vibrations of the laminated plates and shallow shells in Ref [28-39]. Advanced applications of this approach are carried out in Ref [31-33]. The laminated composite shells with layers of variable thickness were considered. Numerical results for cylindrical and spherical shells with complex forms are presented here. In Ref [34-39] the similar approach was applied for forced vibrations of shallow shells of an arbitrary form and some numerical results were presented. In Ref. [29] the R-functions theory has been applied for investigation of nonlinear vibrations of shallow circular cylindrical panels with complex geometry. Lagrange approach is applied in order to reduce the system of motion equations to ordinary differential equations. Multimodal expansions are used and the pseudo arclength continuation method and bifurcation analysis are carried out. Numerical responses are obtained in the spectral neighborhood of the lowest natural frequency.

One of the important applications of the developed approach is in parametric vibrations of the laminated composite plates and shallow shells. Note that the developed software for investigation of parametric vibrations takes into account inhomogeneous subcritical state of the objects and allows finding a buckling load and zones of instability. The main results obtained in this field are presented in Ref. [40-59].

Lately RFM was developed to research nonlinear vibrations of the functionally graded shallow shells. There exist only a few papers, devoted to nonlinear vibrations of FGM shells, Ref. [60-64].

Conclusions

Let us formulate the main problems that should be solved for future development of RFM in order to apply it for investigation of the nonlinear vibrations of the laminated composite and functionally graded shells:

1. To propose, compare and examine another schemes of the discretization of the nonlinear system of motion equationsT.
2. To develop algorithms for the multimode approximation of unknown functions.
3. To investigate nonlinear vibrations of the layered asymmetric shallow shells.
4. To develop methods for study of the parametric vibrations of shell with an arbitrary planform.
5. To create software for investigation of the FGM shells in thermal environment.
6. To work out algorithms for solving nonlinear dynamics problems for plates and shells consisting of FGM and layers of different materials.
7. To improve approximation tools of the undetermined components in solution structures. In order to implement this purpose the appropriate software for splines and orthogonal wavelets on the basis of atomic functions must be worked out.

It is obvious that these problems are challenging. But author dares to hope that they can be successfully solved by the followers of Academician V.Rvachev.

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